

NASA TT F-12,330

THE CAPTURE AND ESCAPE BEHAVIOR OF PLANET
MOON SYSTEMS

O. BSCHORR AND A. LEIBOLD

**CASE FILE
COPY**

Translation of: "Das Einfang- und Katapulvermögen von
Planet/Mond-Systemen"
Deutsche Luft- und Raumfahrt Report 68-81
FB (DVL Report 795), 1968, 44 pages

ABSTRACT: The flyby technique is an effective means of reducing thrust and time of interplanetary missions. The flyby-behavior for planet/moon-systems is investigated in view of flights to planets. The velocity gains and the conditions for capture and escape are calculated. The maximum capture velocity of a Jupiter/Ganymede-flyby is 4.5 km/sec. Probably, the irregular moons of Jupiter, Saturn and Neptune have been captured by a flyby-process.

TABLE OF CONTENTS

SYMBOLS.....	<i>iv</i>
INDICES.....	<i>v</i>
1. INTRODUCTION.....	1
2. PRINCIPLES.....	2
2.1. Hypotheses.....	2
2.2. Orbital Geometry (Fig. 1; Fig. 2).....	3
3. CAPTURE BEHAVIOR OF PLANET/MOON SYSTEMS.....	5
4. MULTI-STAGE CAPTURE MANEUVERS IN THE EARTH/MOON SYSTEM.....	8
5. ESCAPE BEHAVIOR OF THE PLANET/MOON SYSTEM.....	9
6. SUMMARY.....	10
REFERENCES.....	11
APPENDIX.....	13
TABLES.....	14
FIGURES.....	18

SYMBOLS

17

a	= large ellipses (hyperbolic) seminaxis
b	= small ellipses (hyperbolic) seminaxis
\vec{c}	= velocity, referred to Sun
\vec{d}	= rotation vector
\vec{e}	= deviation
\vec{f}	= deviation
$\vec{i}, \vec{j}, \vec{k}$	= unit vector
M	= mass
n	= 1, 2, 3 ...
p	= ellipses (hyperbolic) parameters
r	= radius
R	= radius of Moon
\vec{t}	= rotation vector
u	= circumferential speed
\vec{v}	= velocity, referred to planet
\vec{w}	= velocity, referred to Moon
α	= angle between u_M and v
β	= angle between u_M and w
γ	= gravitation constant
δ	= deflection angle
ϵ	= eccentricity
ϑ	= deflection angle
v	= parameter
φ	= polar angle
τ	= tangent angle
ω	= angle between u_p and v
θ	= stretching versor
Ω	= versor

INDICES

/8

M = Moon
P = Planet
max = maximal
 ∞ = "state prior to entry into gravitational field"
' = state after first flyby
" = state after second flyby
[n] = state after n-th flyby

THE CAPTURE AND ESCAPE BEHAVIOR OF PLANET
MOON SYSTEMS

O. Bschorr and A. Leibold

1. INTRODUCTION

If a spacecraft traverses a gravitational field, it experiences /9* a deflection. Since the gravitational potential is conservative, the relative velocity of the spacecraft to the deflecting body is equal before and after passing through the gravitational field. If the deflecting body has a velocity, e.g. the circumferential velocity of a planet around the Sun, the absolute velocity of the spacecraft results from vector addition of relative and circumferential velocity. Since the vector of relative velocity was rotated, the absolute velocity, referred to the Sun, is generally different before and after the flyby. As is known, very effective trajectory influences can be attained by means of such flyby maneuvers. The flyby maneuvers of planets have been investigated by Von Crocco [1], Hollister [2]-[4], Sohn [5], Niehoff [6], Deerwester [7], Harrison, McLellau [8] and other authors. For example, Hollister has demonstrated that in Mars missions, thrust and flight time could be saved by means of a Venus flyby.

The task of this study is to investigate the effectiveness of flyby maneuvers on the Moons of planets. Such an application appears interesting in the case of Jupiter missions, since it is possible to bring a measuring probe into an orbit around Jupiter without deceleration of the vehicle.

The disadvantage of the flyby technique, namely that the range of possible trajectories and lift-off times is limited since, in addition, a suitable position of the auxiliary planet must be awaited, is not so decisive in the case of maneuvers on moons: since the periods of moons are smaller by the factor 10-100 than in the case of planets; correspondingly, slighter waiting times result.

Moreover, the origin of moon systems is to be discussed with /10 the means of the flyby technique: in continuation of the turbulence theory of v. Weizsäcker [9,10], the regular moons were, according to Kuiper [11], formed from the protoplanetary disc simultaneously with the planets. These moons therefore lie in the equatorial plane with good accuracy. Aside from that, these moons--just as the planets--travel clockwise. Furthermore, circular moon orbits

* Numbers in the margin indicate pagination in the foreign text.

are caused by the mechanism of origination. Friction as opposed to still uncondensed matter and the mass loss function are so-called "resisting means", through which even a previously eccentric orbit becomes circular.

The origin of the Galilean Jupiter moon, i.e. Io, Europa, Ganymede and Callisto, can be explained without contradiction by means of this theory. On the other hand, the remaining moons have very eccentric orbits which are inclined to the Jupiter plane and are, in part, reversed.

Since in the following calculations the capture behavior, e.g. of the Jupiter/Ganymede system, proves to be extraordinarily effective so that bodies with a relative velocity of 4.5 km/sec with regard to Jupiter can still be captured, the capture theory of the irregular moons through a flyby process causes few difficulties; all the more because between the Mars and the Jupiter orbit is the asteroid ring consisting of a large number (40,000) of planetoids. It is more difficult to find a mechanism which so enlarges the orbit of the captured planetoids that these are not catapulted out again by a flyby. The relatively great orbital inclination of these irregular moons also becomes plausible in this view, since an escape in the case of these is very much more improbable.

2. PRINCIPLES

2.1. Hypotheses

A flyby maneuver presents a three-body problem. This is essentially not solvable even for the case that the mass is negligibly small. The following assumptions are made: /11

(a) The "dimensions of the gravitational field" of the deflecting moons are small with respect to the distance to the planet.

(b) The "dimensions of the gravitational field" of the planets are negligibly small with respect to the distance to the Sun.

(c) The distance from planet to moon is to be negligibly small with respect to the Sun to planet distance.

(d) The orbits of the moons are to be circular orbits. This requisite is fulfilled for the regular moons. The eccentric moons are, at any rate, excluded as a result of their slight extrinsic mass from any use as deflecting bodies.

(e) The orbits of the planets are assumed to be circular orbits. This assumption is given in the case of the planets which possess moons.

(f) Orbit of moon and orbit of planet are assumed in numerical

evaluation to be coplanar.

2.2. Orbital Geometry (Fig. 1; Fig. 2)

Probe S moves in an orbit around the Sun. The probe has the velocity \vec{c} and the location vector \vec{r} upon entry into the gravitational field of a planet P. Hypothetically, the "dimensions of the gravitational field" are small with respect to the distance to the Sun, so that the planetary distance \vec{r}_p can be set for the location vector. \vec{c} and \vec{r}_p refer to the Sun.

/12

The planet P moves with velocity \vec{u}_p around the Sun. The relative velocity \vec{v}_∞ of the probe S becomes, with respect to the planet P,

$$\vec{v}_\infty = \vec{c} - \vec{u}_p. \quad (1)$$

With the velocity \vec{v}_∞ and the deviation \vec{e} , the probe enters the planetary gravitational fields and describes a hyperbolic orbit around the planet. This is defined by \vec{v}_∞ and \vec{e} (Fig. 1).

If the probe enters into the gravitational field of the planetary moon M, then the probe will have the velocity \vec{v} and the location vector \vec{r} . Hypothetically, the "dimensions of the gravitational field" of the Moon are small with respect to the distance to the planet so that the distance of the moon can be set as \vec{r}_M for the location vector. \vec{v} and \vec{r}_M refer to the planet P.

A stretching versor θ can be defined, which transforms \vec{v}_∞ into \vec{v} .

$$\vec{v} = \vec{v}_\infty \cdot \theta. \quad (2)$$

θ is defined (appendix) by the stretching ratio

$$\frac{v}{v_\infty} = \sqrt{1 + 2 \left(\frac{u_M}{v_\infty} \right)^2} \quad (3)$$

and the rotation vector \vec{t}

$$\vec{t} = \frac{\vec{e} \times \vec{v}_\infty}{|\vec{e} \times \vec{v}_\infty|} \lg \sqrt{2} \quad (4)$$

The circumferential velocity of the moon M around the planet will be \vec{u}_M . The relative velocity \vec{w}_∞ of the probe is, with respect to the moon (Fig. 2),

-/13

$$\vec{w}_{\infty} = \vec{v} - \vec{u}_M . \quad (5)$$

The probe enters into the gravitational field of the moon with velocity \vec{w}_{∞} and the deviation \vec{f} and describes relative to the moon, a hyperbolic orbit. The probe velocity will be \vec{w}'_{∞} after the flyby of the moon is gravitational field. Since the gravitational potential is conservative, the magnitudes of w_{∞} and w'_{∞} are equal. \vec{w}_{∞} was rotated around the angle δ in \vec{w}'_{∞} .

$$\vec{w}'_{\infty} = \vec{w}_{\infty} \cdot \Omega . \quad (6)$$

One obtains the versor $\cdot \Omega$ from the rotation vector \vec{d} (appendix)

$$\vec{d} = \frac{\vec{f} \times \vec{w}_{\infty}}{|\vec{f} \times \vec{w}_{\infty}|} \operatorname{tg} \delta/2 . \quad (7)$$

The cut-off angle δ is, as is known,

$$\sin \delta/2 = \pm \frac{1}{1 + \frac{r_p w_{\infty}^2}{\gamma M_M}} . \quad (8)$$

It is more expedient to bring equation (8) into the more general forms

$$\sin \delta/2 = \pm \frac{1}{1 + \frac{1}{\gamma^2} \left(\frac{w_{\infty}}{u_M} \right)^2 \left(\frac{r_p}{R} \right)} . \quad (9)$$

The parameter

$$\nu^2 = \frac{\gamma M_M}{R u_M^2} \quad (10)$$

is a value specific for a moon functioning as a deflecting body. One can interpret ν as the ratio of circumferential velocity to orbital velocity. These values for the various moons are tabulated in Table 1.

The probe velocity \vec{v}' referred to the planet is, after leaving /14 the moon's gravitational field,

$$\vec{v}' = \vec{w}'_{\infty} + \vec{u}_M . \quad (11)$$

With Equations (1), (2) and (6):

$$\vec{v}' = [(\vec{c} - \vec{u}_p) \cdot \theta - \vec{u}_M] \cdot \Omega + \vec{u}_M . \quad (12)$$

Under the influence of the planet's gravitational field, the probe describes an orbit which is assigned by the initial conditions \vec{v}' and \vec{r}_M . If the velocity is smaller than the escape velocity, then the probe will remain as an artificial moon in the gravitational range of the planet.

The probe leaves the planet in the case of hyperbolic and parabolic orbits and goes into orbit around the Sun.

If one defines the stretching versor θ' analogously to (2), which transforms \vec{v}' into \vec{v}'_∞ (= velocity of the probe after leaving the planet's gravitational field), then

$$\vec{v}'_\infty = \vec{v}' \cdot \theta' . \quad (13)$$

The velocity \vec{c}' , with which the probe enters into the polar gravitational field, is

$$\vec{c}' = \vec{v}'_\infty + \vec{u}_p . \quad (14)$$

With equations (12), (13) and (14) one obtains the probe velocity \vec{c}' after the flyby maneuver as a function of the initial velocity \vec{c} .

$$\vec{c}' = \{ [(\vec{c} - \vec{u}_p) \cdot \theta - \vec{u}_M] \cdot \Omega + \vec{u}_M \} \cdot \theta' + \vec{u}_p . \quad (15)$$

The location vector of the probe referred to the Sun, is \vec{r}_p . The probe orbit is defined by the initial conditions \vec{c}' and \vec{r}_p . /15

3. CAPTURE BEHAVIOR OF PLANET/MOON SYSTEMS

Especially interesting are those particular approach trajectories which can be captured by planet/moon systems. By means of this, a measuring probe can be brought without deceleration into an orbit around the planet in question.

The probe velocity \vec{v}' after leaving the moon's gravitational field is, after (12)

$$\vec{v}' = [(\vec{c} - \vec{u}_p) \cdot \theta - \vec{u}_M] \cdot \Omega + \vec{u}_M . \quad (12)$$

Here, the probe is at distance r_M from the planet. If v' is smaller than the escape velocity v_{escape}

$$v_{\text{escape}} = \sqrt{\frac{2\gamma M_p}{r_M}} = u_M \sqrt{2}, \quad (16)$$

then the probe remains as an artificial moon in the gravitational range of the approached planet. The capture condition is given by (12) and (16):

$$u_M \sqrt{2} \geq |[(\vec{c} - \vec{u}_p) \cdot \vec{Q} - \vec{u}_M] \cdot \vec{\Omega} + \vec{u}_M|. \quad (17)$$

With restriction to the plane case, evaluation of (17) yields

$$\left(\frac{v}{u_M}\right)^2 \leq \left(\frac{w_\infty}{u_M}\right)^2 - 2\left(\frac{w_\infty}{u_M}\right) \cos(\beta' - \delta) - 1 \quad (18)$$

$$\left(\frac{v_{\text{max}}}{u_M}\right)^2 = \left(\frac{w_\infty}{u_M}\right)^2 - 2\left(\frac{w_\infty}{u_M}\right) \cos(\beta' - \delta) - 1.$$

Set here is

/16

$$\sin \delta/2 = \pm \frac{1}{1 + \frac{1}{v^2} \left(\frac{w_\infty}{u_M}\right)^2 \left(\frac{r_0}{R}\right)}, \quad (9)$$

$$\cos \beta' = \frac{1}{2} \left(\frac{w_\infty}{u_M} - \frac{u_M}{w_\infty} \right). \quad (19)$$

v resp. v_{max} is the probe velocity at distance r_M from the planet. The probe velocity v_∞ resp. $v_{\infty \text{ max}}$ upon entry into the planet's gravitational field is, after the energy relation

$$\begin{aligned} v_\infty &= \sqrt{v^2 - 2u_M^2}, \\ v_{\infty \text{ max}} &= \sqrt{v_{\text{max}}^2 - 2u_M^2}. \end{aligned} \quad (20)$$

Equations (18), (9), (19) and (20) were evaluated in order to obtain the highest possible relative velocity $v_{\infty \text{ max}}$, at which a capture of the probe by a planet/moon flyby is just possible. It is shown that the maximum for

$$r_0 = R$$

is present when the apical distance r_0 assumes, in the flyby of the

moon (compare Fig. 2), the smallest possible value.

The results of the optimization calculation are compiled in Table 3 for the various moon systems. The numerically greatest capture velocity results in the Jupiter/Ganymede system with 4.5 km/sec. Probes with a smaller relative velocity with respect to Jupiter can be captured by Jupiter as artificial moons in this way without deceleration.

The velocity conditions in the optimal capture process are presented in Figure 3.

After Equation (3)

$$\vec{c}_{max} = \vec{v}_{\infty max} + \vec{u}_p \quad (3')$$

the velocity vector \vec{c}_{max} and with it the data of the captured paths are given with known $v_{\infty max}$. Since the moon rotates around the planet and thus assumes all angles from 0° - 360° with respect to \vec{u}_p , $\vec{v}_{\infty max}$ can also have all directions ω . /17

The parameters p and ϵ of the capturable trajectories are defined by the conditions (p and ϵ referred to a sun-based coordinate system):

$$\frac{p}{r_p} = \left(1 - \frac{v_{\infty}}{u_p} \cos \omega\right)^2, \quad (21)$$

$$\epsilon = \frac{c_m}{u_p} \sqrt{\left(1 - \frac{v_{\infty}}{u_p} \cos \omega\right)^2 + 3 \cos^2 \omega - 2 \frac{v_{\infty}}{u_p} \cos^3 \omega}, \quad (22)$$

$$0 \leq v_{\infty} \leq v_{\infty max}, \quad (23)$$

$$0^\circ \leq \omega \leq 360^\circ. \quad (24)$$

These fields are presented for the Earth's Moon in Figure 4, for the moons of Jupiter in Figure 5 and for Saturn's moon Titan in Figure 6. The other moons were not considered because of their low, and, in part, unknown intrinsic mass.

It is shown that the range of the trajectories capturable by a planet/moon flyby maneuver is relatively large. Especially the Jupiter/Jupiter-moon systems cover a range which extends far into the planetoid ring. The probabilities that a planetoid body will be coincidentally captured in a flyby procedure at one of the

Galilean moons cannot be excluded. While the largest Galilean moons originated simultaneously with Jupiter from the protoplanetary mass, the remaining Jupiter moons were very probably captured in this way. Since, conversely, a body can also be catapulted out again by a flyby maneuver, the only moons capable of remaining Jupiter's field are those whose trajectories fulfill special conditions of stability. A similar stabilization effect is imaginable in the Jupiter/Jupiter-moon system, analogous to the known commensurability gaps and commensurability frequencies in the planetoid system. This presumption /18 is supported by the fact that the clockwise moons VI, VII and X present approximately the same period (250-260 days), and the counter-clockwise moons VIII, IX, XI and XII likewise have approximately equal period (625-760 days).

4. MULTI-STAGE CAPTURE MANEUVERS IN THE EARTH/MOON SYSTEM

Success can be met by means of a one time flyby maneuver in the Earth/Moon system in capturing bodies which possess a velocity $v_{\infty \max} = 1.85$ km/sec with respect to the Earth's orbit [12]. If the body is faster than the critical velocity of 1.85 km/sec, it loses velocity through a flyby maneuver; it leaves, however, the Earth's range of attraction and again goes into orbit around the Sun. This path and the Earth's orbit have common points of intersection; and it is possible to perform a second flyby maneuver of the moon during coincidence.

This procedure can be repeated as often as desired until the velocity of the bodies falls below the critical capture velocity and it can be captured.

The calculation of multi-stage flyby maneuvers is connected with the basic equation (15).

$$\vec{c}' = \{[(\vec{c} - \vec{u}_p) \cdot \vec{\theta} - \vec{u}_M] \cdot \Omega + \vec{u}_M\} \cdot \vec{\theta}' + \vec{u}_p. \quad (15)$$

The probe has the velocity \vec{c}' after the flyby maneuver, referred to the Sun, and orbits the Sun as an artificial planet. In the course of this, the magnitude and direction of the probe velocity is altered in accordance with the laws of Kepler. If the probe trajectory again intersects the Earth's orbit, then the probe has the velocity \vec{c}' . Thus, \vec{c}' is the new entry velocity in the case of a repeated flyby maneuver, and, analogous to equation (15), the exit velocity \vec{c}'' results after the second flyby:

$$\vec{c}'' = \{[(\vec{c}' - \vec{u}_p) \cdot \vec{\theta}' - \vec{u}_M] \cdot \Omega' + \vec{u}_M\} \cdot \vec{\theta}'' + \vec{u}_p. \quad (25)$$

Correspondingly, for the n-th flyby maneuver:

/19

$$\vec{c}^{(n)} = \{ [(\vec{c}^{(n-1)} \hat{u}_p) \cdot \theta^{(n-1)} \hat{u}_M] \cdot \Omega^{(n-1)} \hat{u}_M \} \cdot \theta^{(n)} + \hat{u}_p . \quad (26)$$

In order here to attain maximal deceleration, the versors θ and Ω must be optimized in such a way that

$$c^{(n-1)} - c^{(n)} \rightarrow \text{Maximum} . \quad (27)$$

With limitation to the coplanar case, the optimization calculation for an n-stage flyby maneuver yields the deceleration velocities indicated in Table 2. The capture boundaries are presented in Figure 4. It is shown that multiple repetition of a moon flyby procedure is not effective. Aside from that, such maneuvers are very time-consuming, since, in each case according to the commensurability relation of the periods of the Earth and of the probe, an encounter takes place again only after several orbits.

5. ESCAPE BEHAVIOR OF THE PLANET/MOON SYSTEM

It is possible in a reversal of the capture maneuver to catapult a probe out of a planet's range of attraction by means of a moon flyby. Such a case of application exists when, for example, a Jupiter probe is to be brought back again to or into the vicinity of the Earth after termination of the measurement mission. A direct transcription of the test data requires extremely great transmission powers because of the great distance to Jupiter.

The escape process is likewise described by the general flyby equation (15). The solar orbits attainable by means of an escape process correspond to those which can be captured by a capture maneuver by the planet/moon system. These trajectories are presented for the various moon systems in Figures 4-6. It is also interesting to determine those particular orbits around a planet which can be catapulted out by means of a flyby maneuver of a planetary moon. The following derivation results for this case:

/20

A probe path around the planet will be given by the velocity \vec{v} and the rotation vector \vec{r}_M . \vec{v} and \vec{r}_M will refer to the planet. The circumferential velocity of the moon will be \vec{u}_M . The probe will thus enter into the moon's gravitational field with the velocity \vec{w}_∞

$$\vec{w}_\infty = \vec{v} - \vec{u}_M . \quad (5)$$

\vec{w}_∞ is rotated in the moon's gravitational field into \vec{w}'_∞

$$\vec{w}' = \vec{w}_\omega \cdot \Omega . \quad (6)$$

The velocity \vec{v}' , referred to the planet, after the moon flyby, is

$$\vec{v}' = \vec{w}' + \vec{u}_M . \quad (11)$$

Equations (5), (6) and (11) yield end velocity \vec{v}' as a function of the initial velocity \vec{v} .

$$\vec{v}' = (\vec{v} + \vec{u}_M) \cdot \Omega + \vec{u}_M . \quad (28)$$

If \vec{v}' is larger than the escape velocity v_{escape}

$$v_{\text{escape}} = u_M \sqrt{2} , \quad (16)$$

then the probe leaves the range of attraction of the planet. The escape condition is

$$u_M \sqrt{2} \leq |(\vec{v} + \vec{u}_M) \cdot \Omega + \vec{u}_M| . \quad (29)$$

This relation was evaluated for the various planet/moon systems /21 for the coplanar case. Those particular moon paths which can be catapulted out of the range of attraction of the planet in question by a flyby process are presented in Figures 7-13. It is shown, for example, that in the Jupiter systems only very nearly parabolic trajectories can escape by means of a flyby process, in spite of the great intrinsic masses of the Galileo moons. The criterion for the escape behavior of a planet/moon system is the v -value (Table 1).

The masses of the individual moons, are known only very imprecisely. That is a result of the fact that they can only be calculated indirectly from trajectory disturbances.

In the case of a moon flyby by a measuring probe, the trajectory disturbance is very much larger, and thus the moon mass can be determined more precisely. The most precise method is to bring the probe into an orbit around the moon.

6. SUMMARY

As is known, a spacecraft can be brought without thrust from a low-energy to a high-energy flight trajectory by means of a flyby

maneuver, and conversely. A series of numerical investigations have been published for flybys of planets [1]-[8]. A multi-stage flyby mission of Jupiter, Saturn, Uranus and Neptune has been calculated by Flandro [3].

/22

The effectiveness of flyby maneuvers of planetary moons is investigated in the present study.

In particular, the capture behavior of a planet/moon system is ascertained. Such a technique appears interesting for Jupiter missions, to bring a measuring probe into orbit around Jupiter without deceleration on the part of the spacecraft. The maximal capture velocity of the Jupiter/Ganymede system amounts to 4.5 km/sec.

Also, multi-stage flyby maneuvers are investigated for the Earth/Moon system. The maximal capture velocity is 1.85 km/sec with a single stage maneuver; with a two-stage, it is 2.5 km/sec. With stage number becoming larger, the yield attainable per stage becomes slighter.

It can be made probable that the so-called irregular Jupiter, Saturn and Neptune moons were captured by a flyby process of a regular planetary moon.

In a reversal of the capture behavior, the catapult behavior of planet/planetary moon systems is indicated.

The moon flyby technique offers the possibility of determining moon masses with greater exactness.

REFERENCES

/23

1. Crocco, G.A.: One Year Exploration Trip. Proceedings of the VIIth Int. Astronaut. Congr. 1956.
2. Hollister, W.M.: The Mission for a Manned Expedition to Mars. Sc. D. Thesis M.I.T. Cambridge 1963.
3. Hollister, W.M.: Mars Transfer via Venus. AIAA/ION Astrodynamics Guidance and Control Conf. Los Angeles 1964.
4. Hollister, W.M., and I.E. Prussing: Optimum Transfer to Mars via Venus. Astronautica Acta, Vol. 12, No. 2, 1966.
5. Sohn, R.L.: Venus Swingby Mode for Manned Mars Missions. J. Spacecraft and Rockets, Vol. 1, No. 5, 1964.
6. Niehoff, J.C.: Gravity-Assisted Trajectories to Solar-System Targets. J. Spacecraft and Rockets, Vol. 3, No. 9, 1966.
7. Deerwester, J.M.: Jupiter Swingby-Missions to Outer Planets. AIAA Paper 66-536.
8. Harrison, E.F., and C.H. McLellau: Analysis of Use of Jupiter Gravity Turns for Tailoring Trajectories to Saturn with Consideration of Launch Opportunities. NASA TN D-3699, Nov. 1966.

9. Von Weizsäcker, C.F.: Entstehung des Planetensystems. (Genesis of the Planetary System) Z. Astrophysik, No. 5, 1943.
10. Von Weizsäcker, C.F.: Zur Kosmogonie. (On Cosmogony) Z. Physik, pp. 181-206, 1947.
11. Kuiper, G.P.: The Formation of the Planets. J. of the Royal Astronomical Society of Canada, Vol. 50, No. 2; No. 3; No. 4 124 1955.
12. Bschorr, O.: Bahntransitionen in bewegten Schwerfeldern. (Trajectory Transition in Moving Gravitational Field) WGLR-Jahrbuch 1966.
13. Flandro, G.A.: Fast Reconnaissance to the Outer Solar System Utilizing Energy Derived from Gravitational Field of Jupiter. Astronautica Acta, Vol. 12, No. 4, 1966.
14. Deerwister, J.M., and S.M. D'Haem: Systematic Comparison of Venus Swingby Mode with Standard Mode of Mars Round Trip J. Spacecraft and Rockets, Vol. 4, No. 7, 1967.
15. Jones, A.: Solar System Escape Circle. J. Spacecraft and Rockets Vol. 4, No. 7, 1967.
16. Battin, R.H.: Astronautical Guidance. New York: McGraw Hill, 1964.
17. Lee, V.A. et al: Trajectory and Mission Analysis Aspects of Jupiter Flyby Probes. 18. Int. Astronautical Congress. Belgrad, Sept. 1967.
18. Bschorr, O.: Gewinnung von Rohstoffen aus außerirdischen Lagerstätten. (Obtaining Crude Materials from Extraterrestrial Deposits) Deutsche Auslegungsschrift Vol. 1, p. 229, p. 969, 1961.
19. Lagally, M.: Vorlesungen über Vektorrechnung. (Lectures on Vector Calculation) Leipzig: Akademische Verlagsges. Geest und Portig KG, 1949.

Translated for the National Aeronautics and Space Administration by:
 Aztec School of Languages, Inc.,
 Research Translation Division (524)
 Maynard, Massachusetts.
 NASw-1692.

Formulas for the establishment of the versor from axis of rotation and angle of rotation.

The direction of the axis of rotation will be given by the unit vector \vec{n} :

$$\vec{n} = \cos \alpha \vec{i} + \cos \beta \vec{j} + \cos \gamma \vec{k}$$

($\cos \alpha$, $\cos \beta$ and $\cos \gamma$ are the cosines of direction)

The angle of rotation around the axis \vec{n} will be δ . From that results the rotation vector \vec{d}

$$\vec{d} = \operatorname{tg} \delta/2 \vec{n}$$

$$\vec{d} = a \vec{i} + b \vec{j} + c \vec{k}.$$

Set here is:

$$\begin{aligned} a &= \cos \alpha \operatorname{tg} \delta/2 \\ b &= \cos \beta \operatorname{tg} \delta/2 \\ c &= \cos \gamma \operatorname{tg} \delta/2. \end{aligned}$$

Thus, the versor Ω results after Lagally [19] at

$$\Omega = \frac{1}{1+a^2+b^2+c^2} \begin{bmatrix} 1+a^2-b^2-c^2 & 2(ab+c) & 2(ac-b) \\ 2(ab-c) & 1-a^2+b^2-c^2 & 2(bc+a) \\ 2(ac+b) & 2(bc-a) & 1-a^2-b^2+c^2 \end{bmatrix}.$$

Reduced for the plane case with rotation around the k-axis:

$$\Omega = \begin{bmatrix} \cos \delta & \sin \delta & 0 \\ -\sin \delta & \cos \delta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

TABLE 1. DATA ON MOONS

Planet Moon	Mean Distance From Planet 10^3 km	Moon's Mass g	Moon's Radius km	Mean Circumferential Velocity km/sec	v^2 [$\frac{1}{2}$]	Trajectory Inclination [$^\circ$]
Earth Moon	384		1738	1.02	2.71	
Mars Phobos Deimos	9.4 23.5	- -	8 4.5	2.14 1.36	- -	
Jupiter V I Io II Europa III Ganymede IV Callisto VI VII X XII XI VIII IX	181 421 670 1069 1881 11450 11740 11750 21000 22550 23500 23950	79.10 ²⁴ 47.10 ²⁴ 135.10 ²⁴ 90.10 ²⁴ - - - - - -	1960 1680 2755 2525 - - - - - -	26.3 17.3 13.7 10.9 8.2 3.3 3.285 3.28 2.45 2.40 2.32 2.30	- 0.01056 0.01174 0.0347 0.0313 - - - - - -	3 ⁰⁷ Counter- clock- wise

TABLE 1. (CONT'D)

Planet Moon	Mean Distance From Planet 10^3 km	Moon's Mass g	Moon's Radius km	Mean Circumferen- tial Velocity km/sec	v^2 [-7]	Trajec- tory Inclin- ation [$]^0$
Saturn						
Mimas	185	-	250	14.3	-	
Enceladus	238	-	300	12.7	-	
Thetis	295	-	600	11.4	-	
Dione	377	-	650	10.0	-	
Rhea	527	-	900	8.5	-	
Titan	1221	$137 \cdot 10^{24}$	2500	5.58	0.1172	$26^{\circ}45'$
Hyperion	1482	-	-	5.05	-	
Japetus	3558	-	600	3.26	-	
Phoebe	12946	-	-	1.71	-	
Uranus						
Miranda	120	-	-	6.7	-	
Ariel	192	-	-	5.5	-	97°
Umbriel	267	-	-	4.7	-	
Titania	438	-	500	3.7	-	
Oberon	586	-	400	3.2	-	
Neptune						
Triton	355	$150 \cdot 10^{24}$	2000	4.39	0.268	Counter- clock- wise
Nereide	6000	-	-	-	-	

TABLE 2. MULTI-STAGE FLYBY MANEUVERS IN THE EARTH/MOON SYSTEM

Number of Flyby Maneuvers n		1	2	3	4	5	6	7	8	9	10
Entry Velocity v [n]	km/sec	1.85	2.58	3.09	3.51	3.85	4.15	4.42	4.67	4.89	5.09
Exit Velocity v' [n]	km/sec	0	1.85	2.58	3.09	3.51	3.85	4.15	4.42	4.67	4.89
Deceleration $v' - v$ [n]	km/sec	1.85	0.63	0.51	0.42	0.34	0.30	0.27	0.25	0.22	0.20

TABLE 3. MAXIMAL CAPTURE VELOCITIES IN PLANET/MOON FLYBY.

<i>Planet</i>	MOON	MAXIMAL CAPTURE VELOCITY $v_{\infty \max}$ [KM/SEC]
EARTH	MOON	1.85 KM/SEC
<i>Jupiter</i>	<i>Io</i>	4.13 "
	<i>Europa</i>	3.44 "
	<i>Ganymede</i>	4.49 "
	<i>Callisto</i>	3.23 "
<i>Saturn</i>	<i>Titan</i>	3.76 "
NEPTUNE	<i>Triton</i>	4.02 KM/SEC

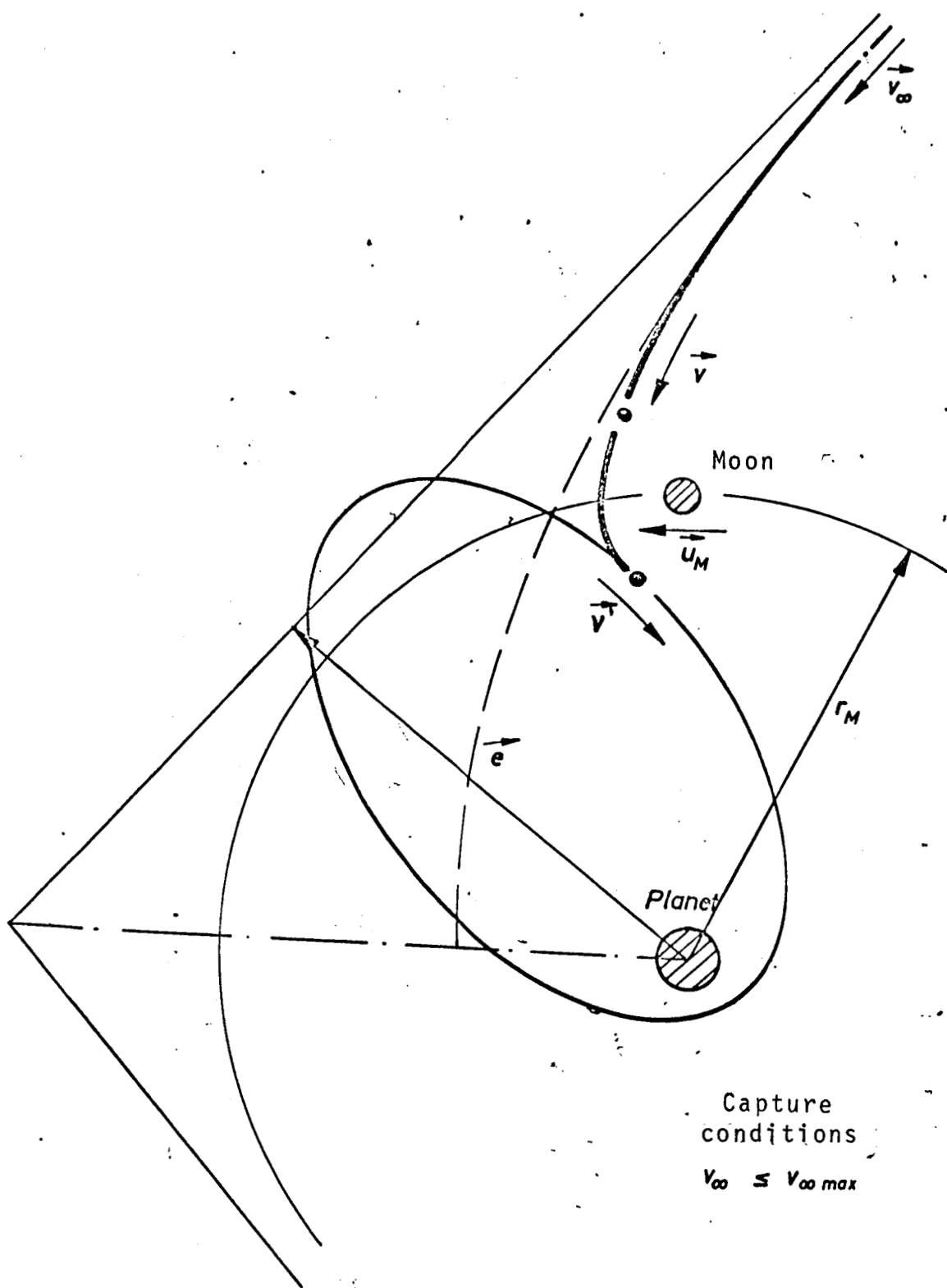


Fig. 1. Trajectory Relations in a Planet-Based Coordinate System.

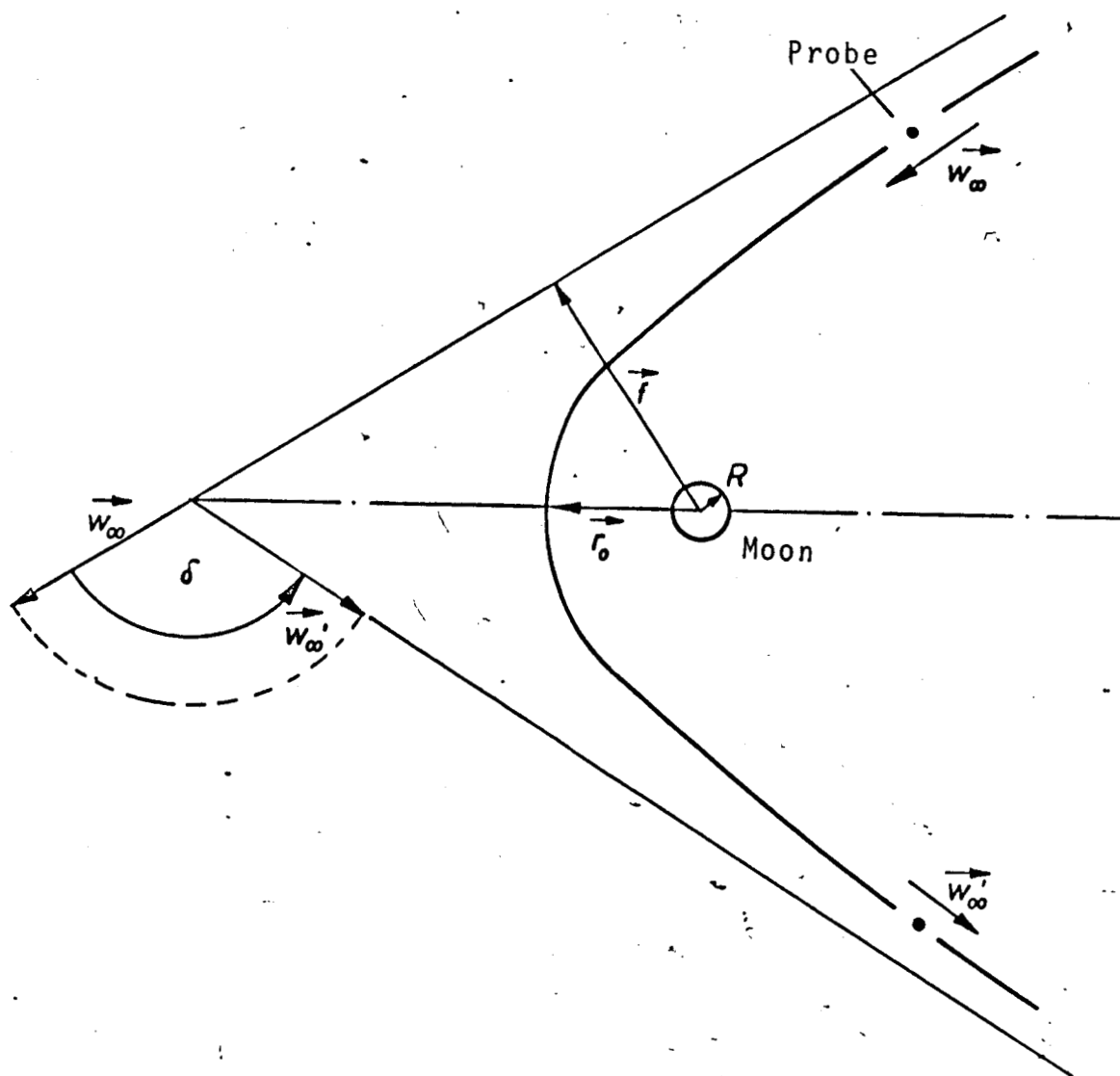


Fig. 2. Trajectory Relations in a Moon-Based Coordinate System.

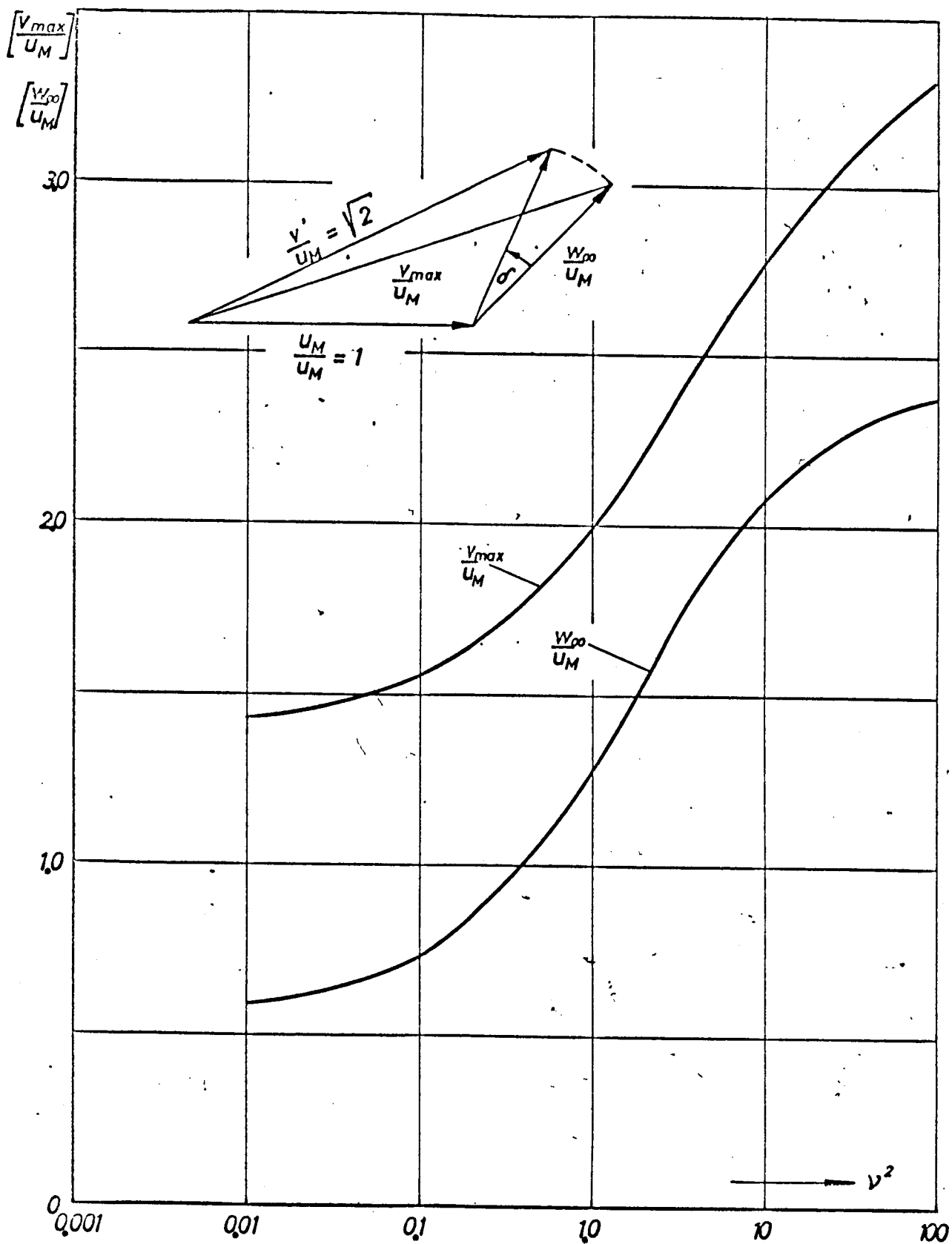


Fig. 3. Velocity Relations in Capture Process.

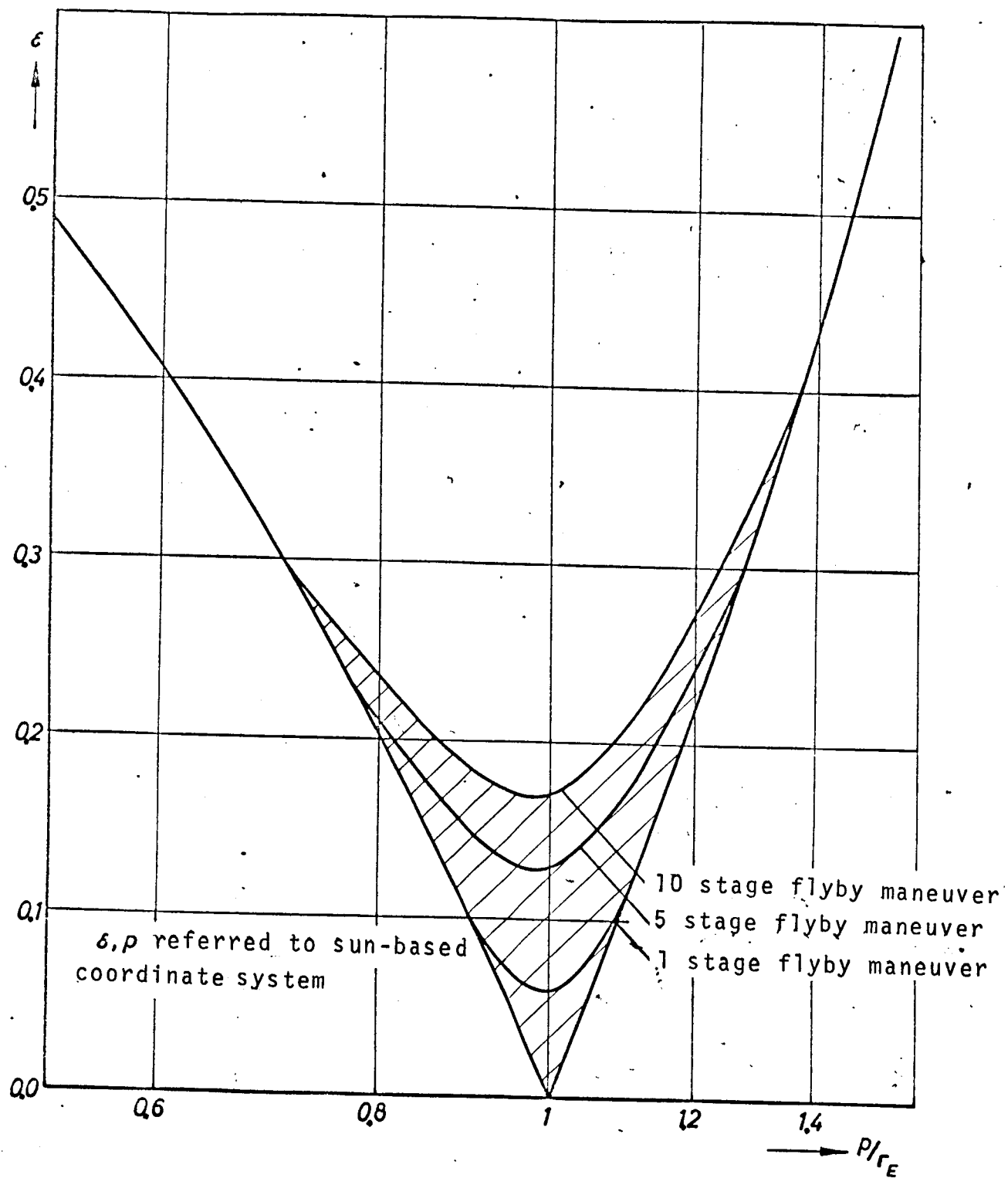


Fig. 4. Capture Behavior of the Earth/Moon System.

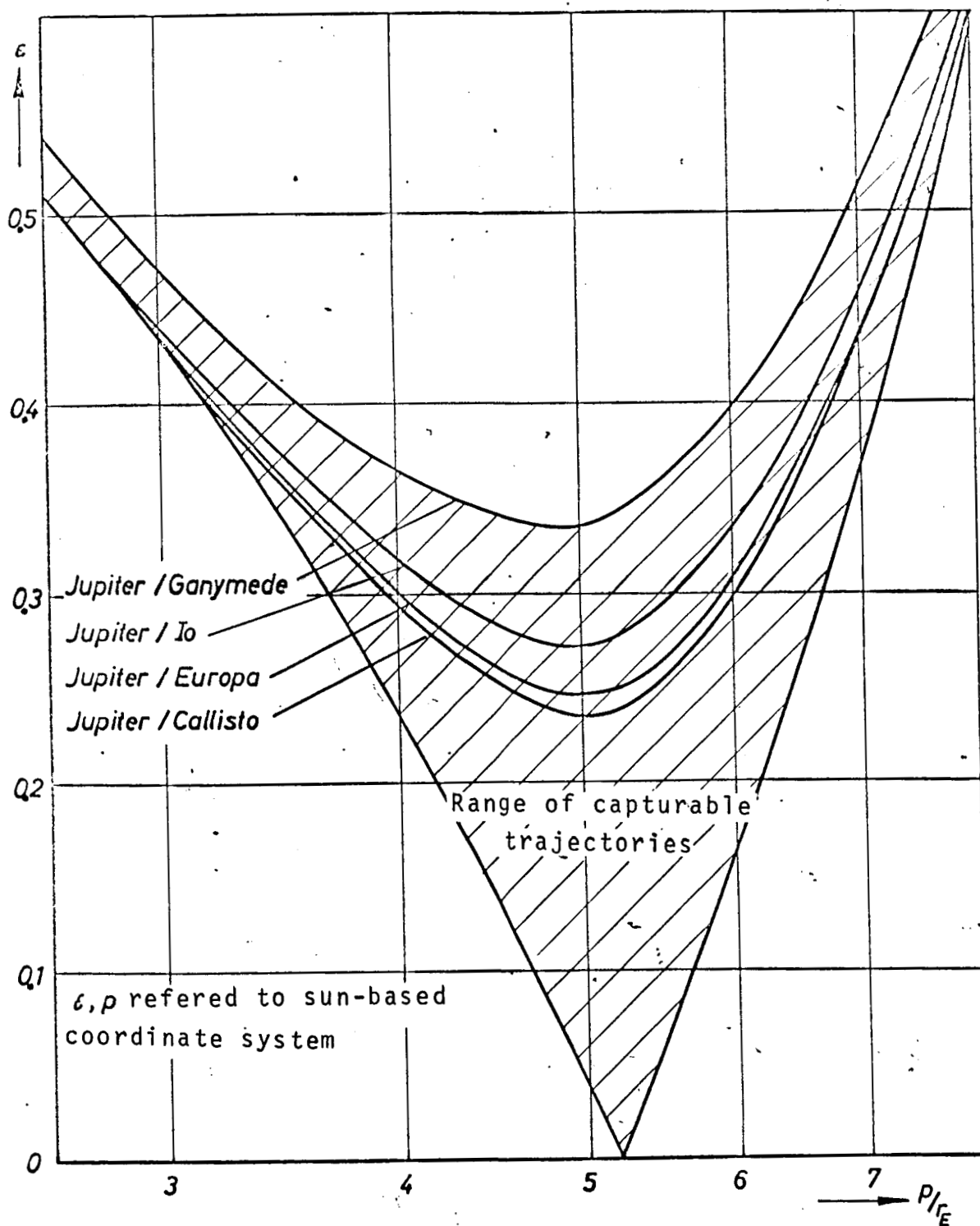


Fig. 5. Capture Behavior of the Jupiter/Jupiter Moon System.

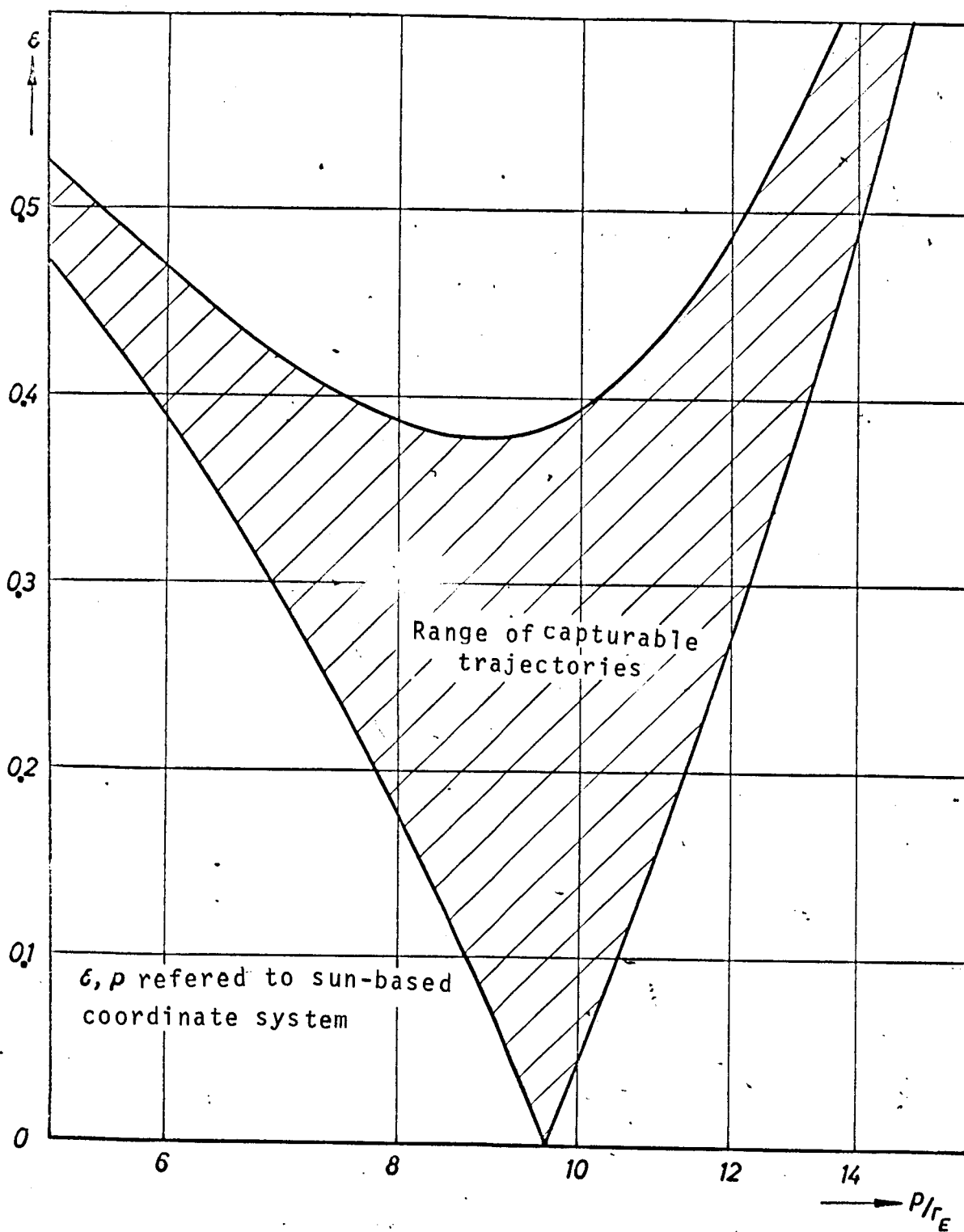


Fig. 6. Capture Behavior of the Saturn/Titan System

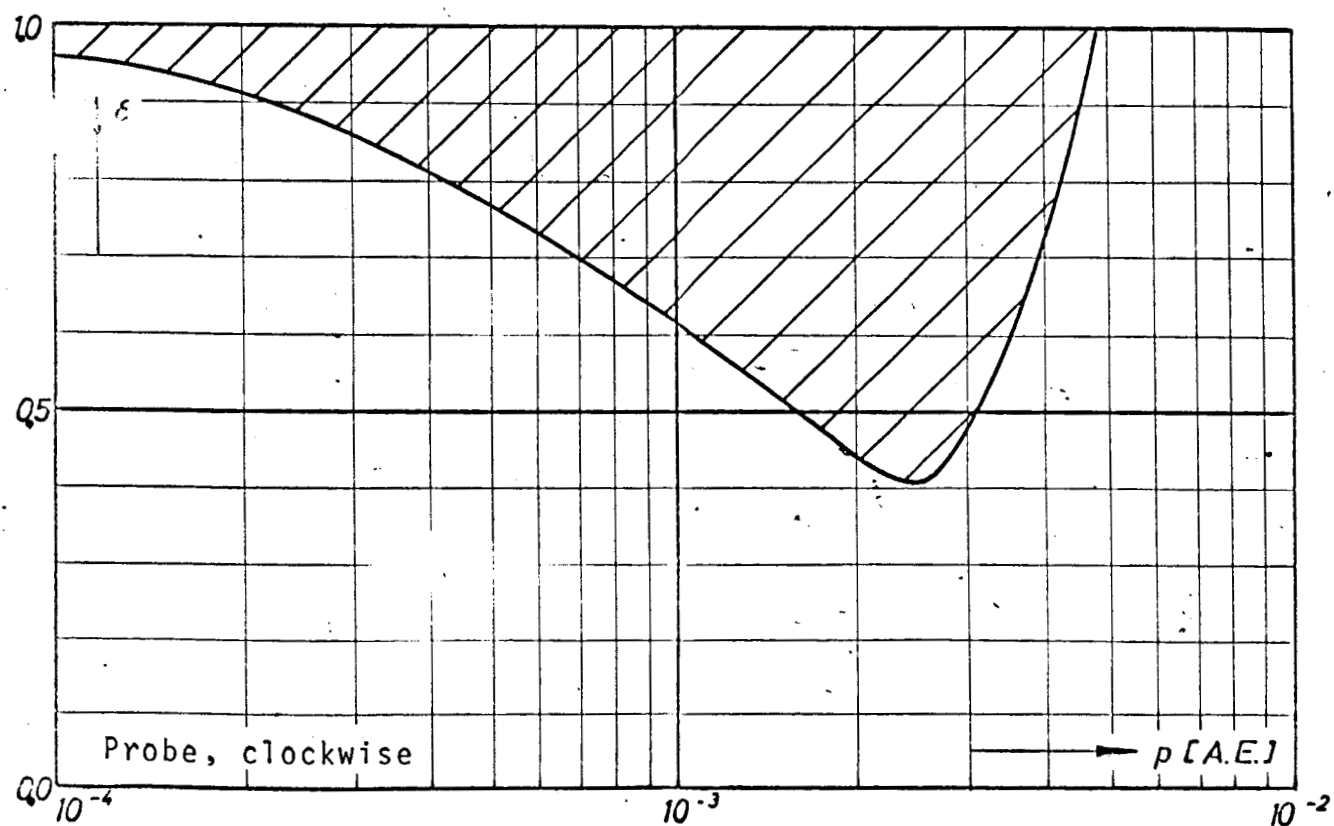
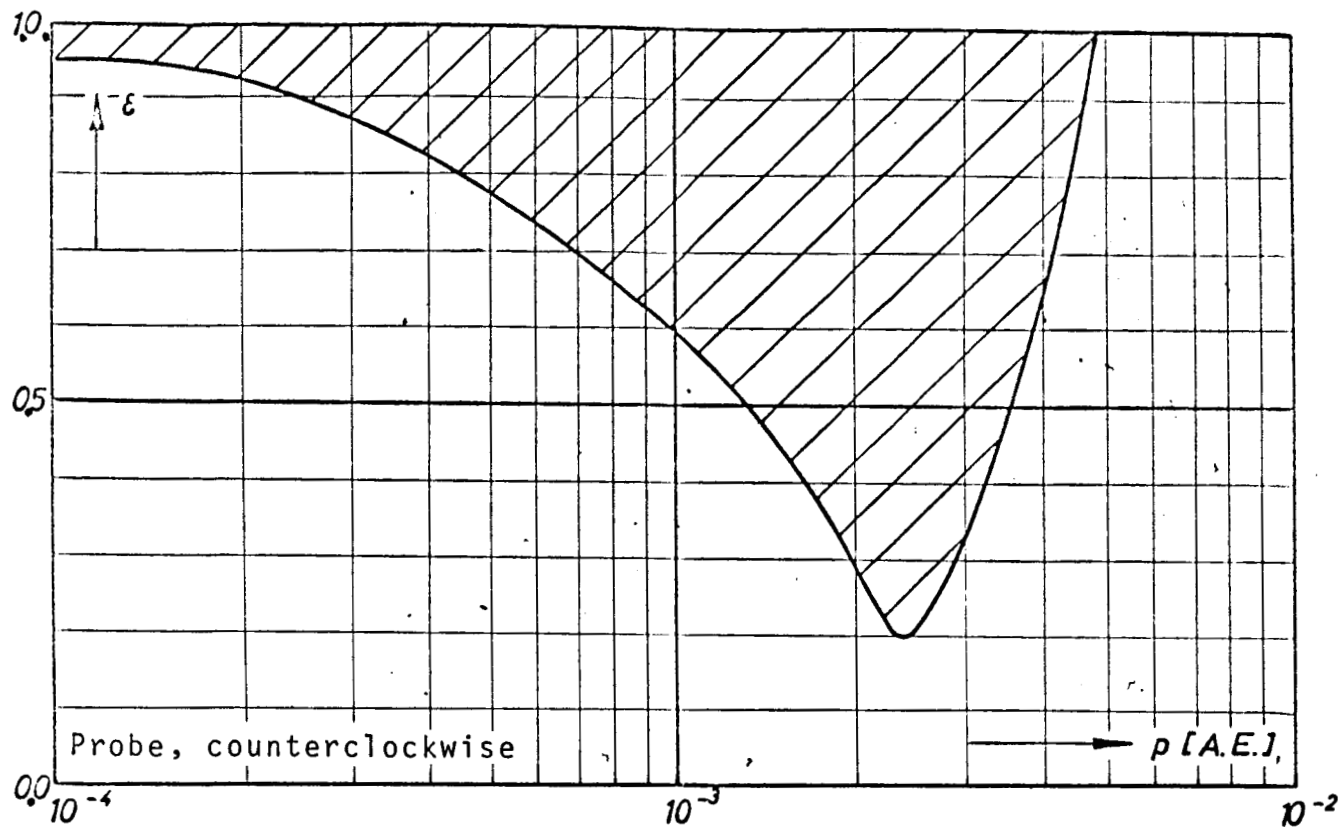


Fig. 7. Escape Behavior of the Earth/Moon System.

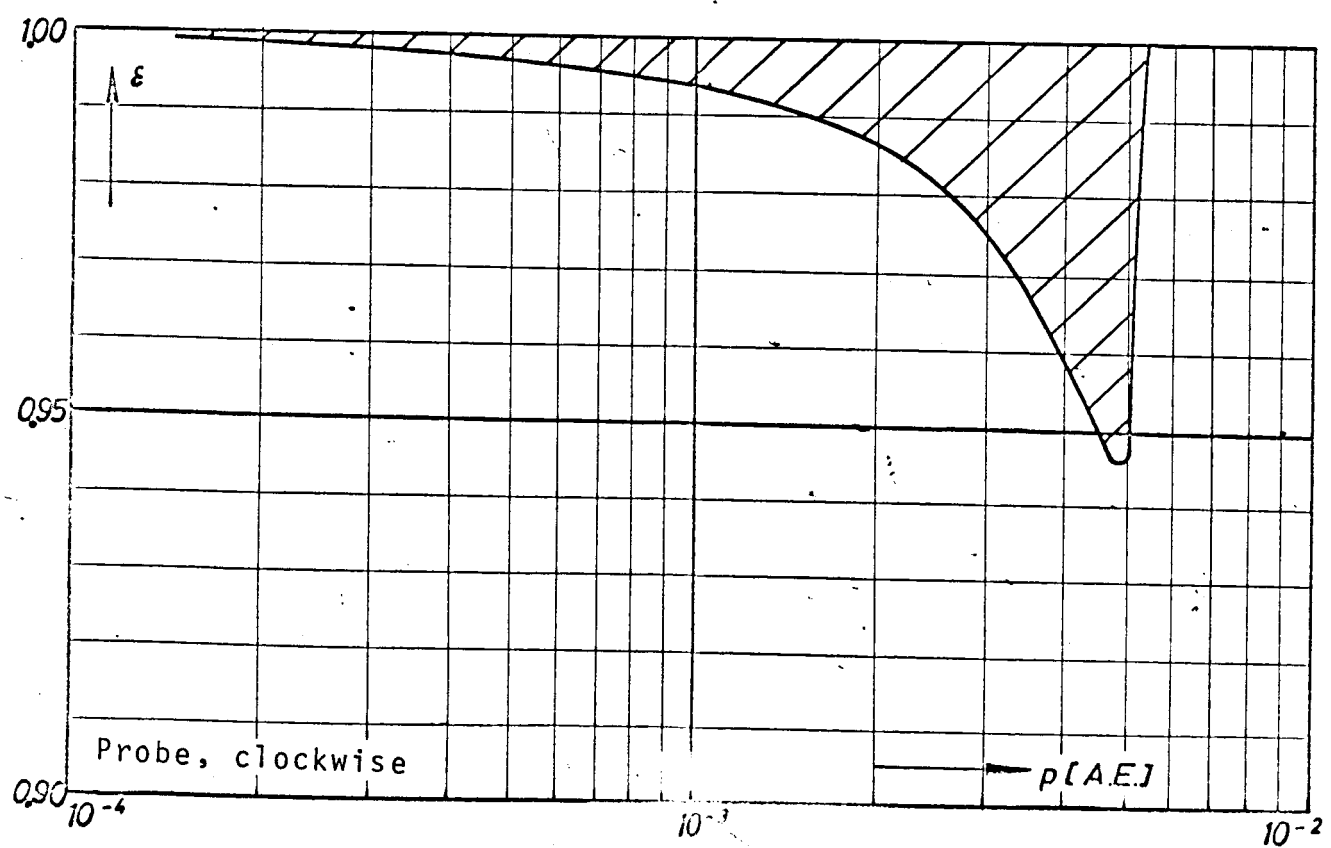
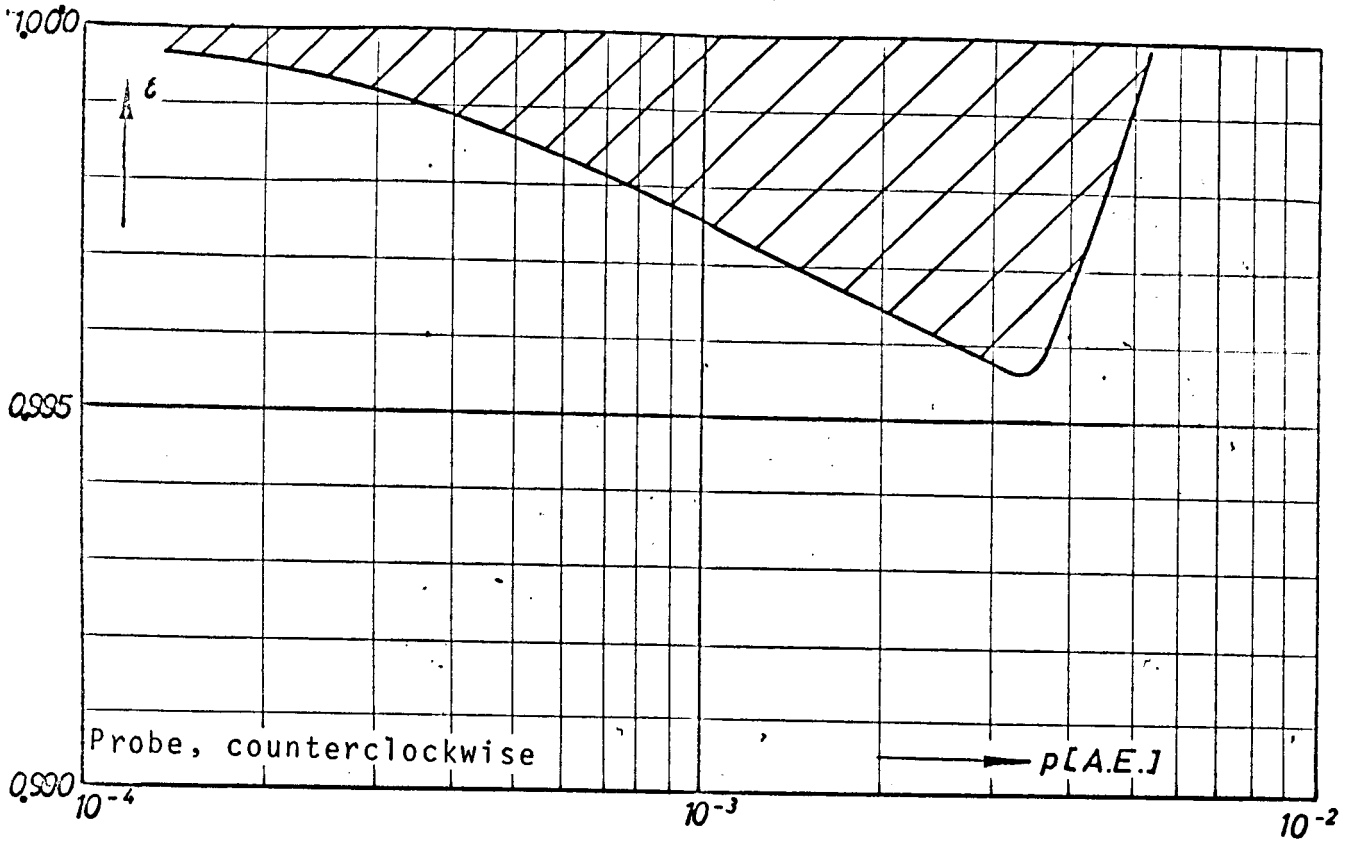


Fig. 8. Escape Behavior of the Jupiter/Io System.

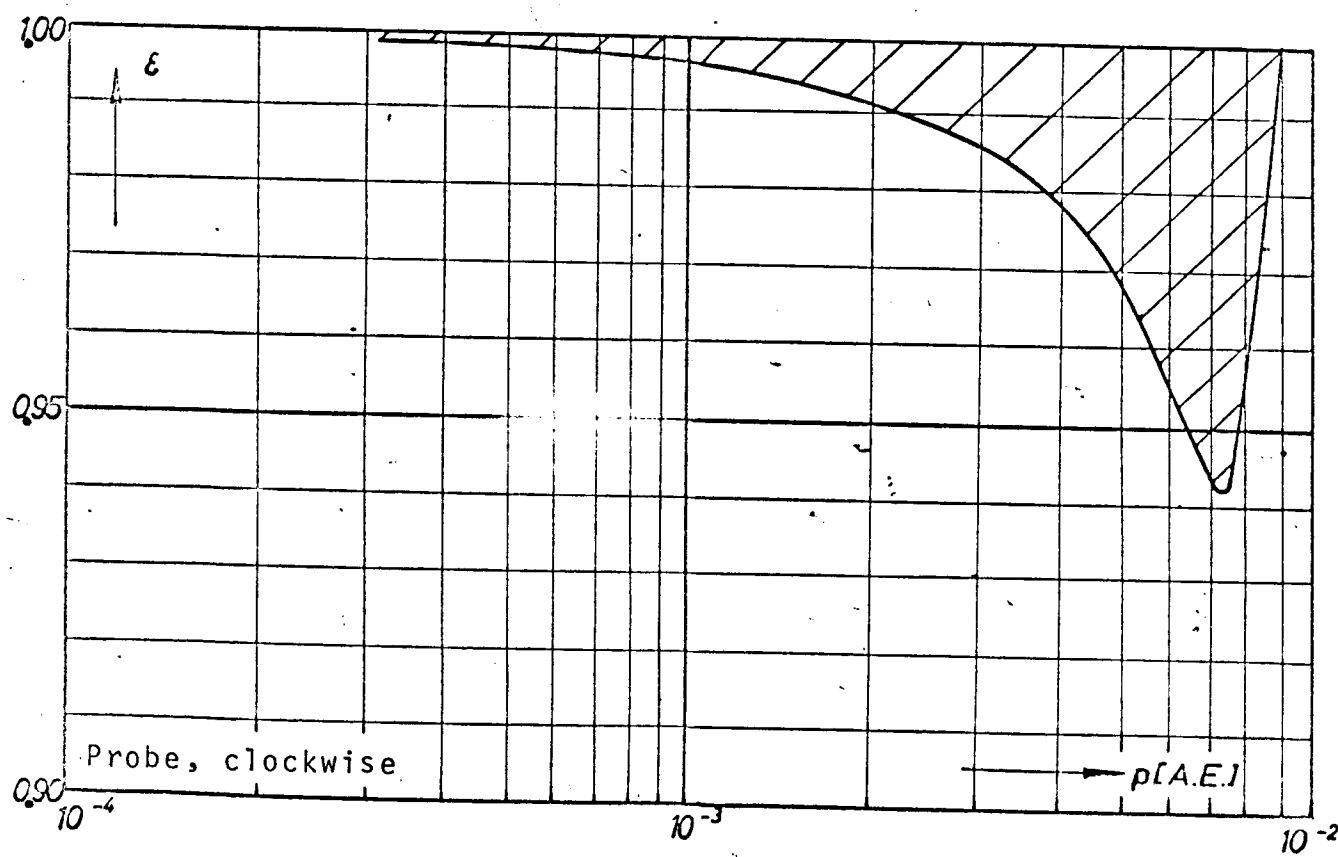
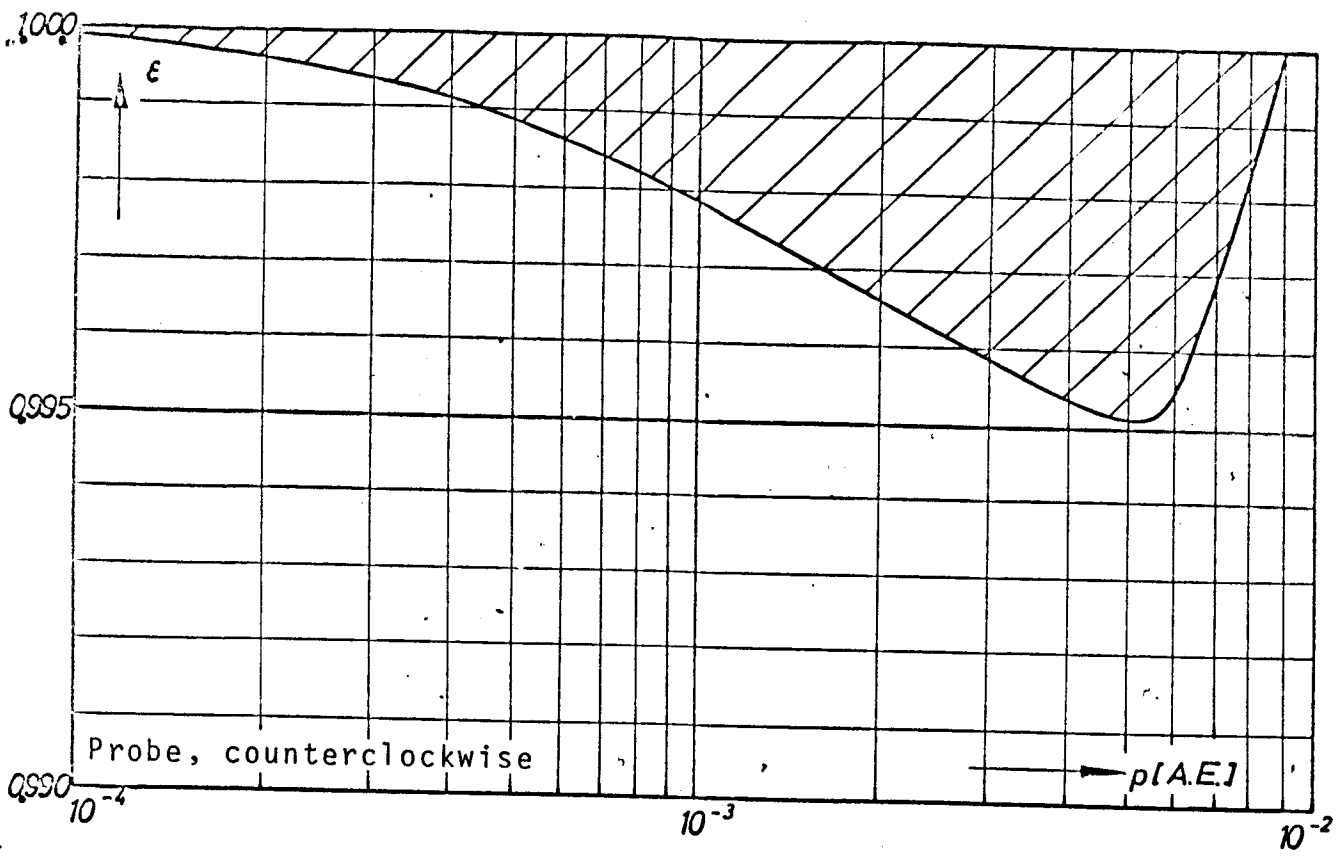


Fig. 9. Escape Behavior of the Jupiter/Europa Systems.

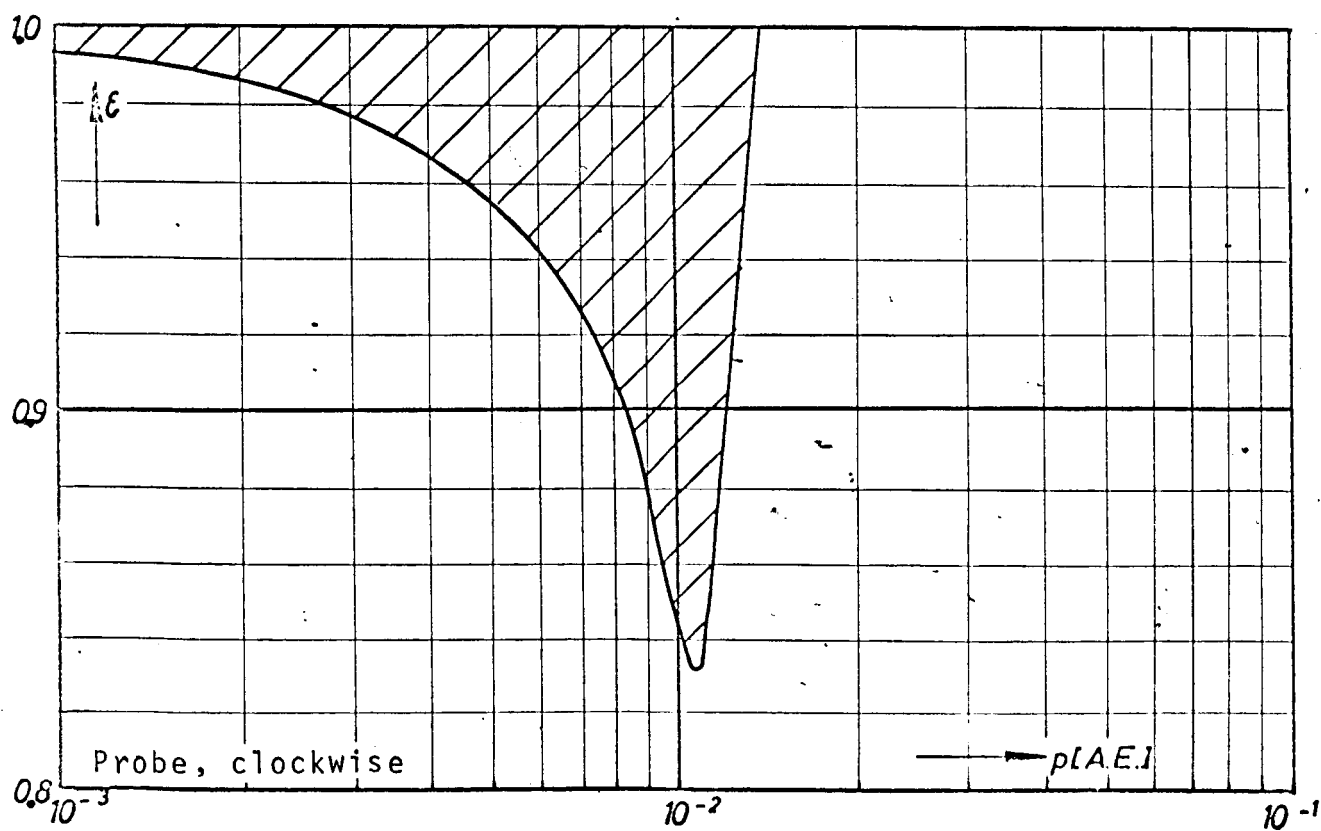
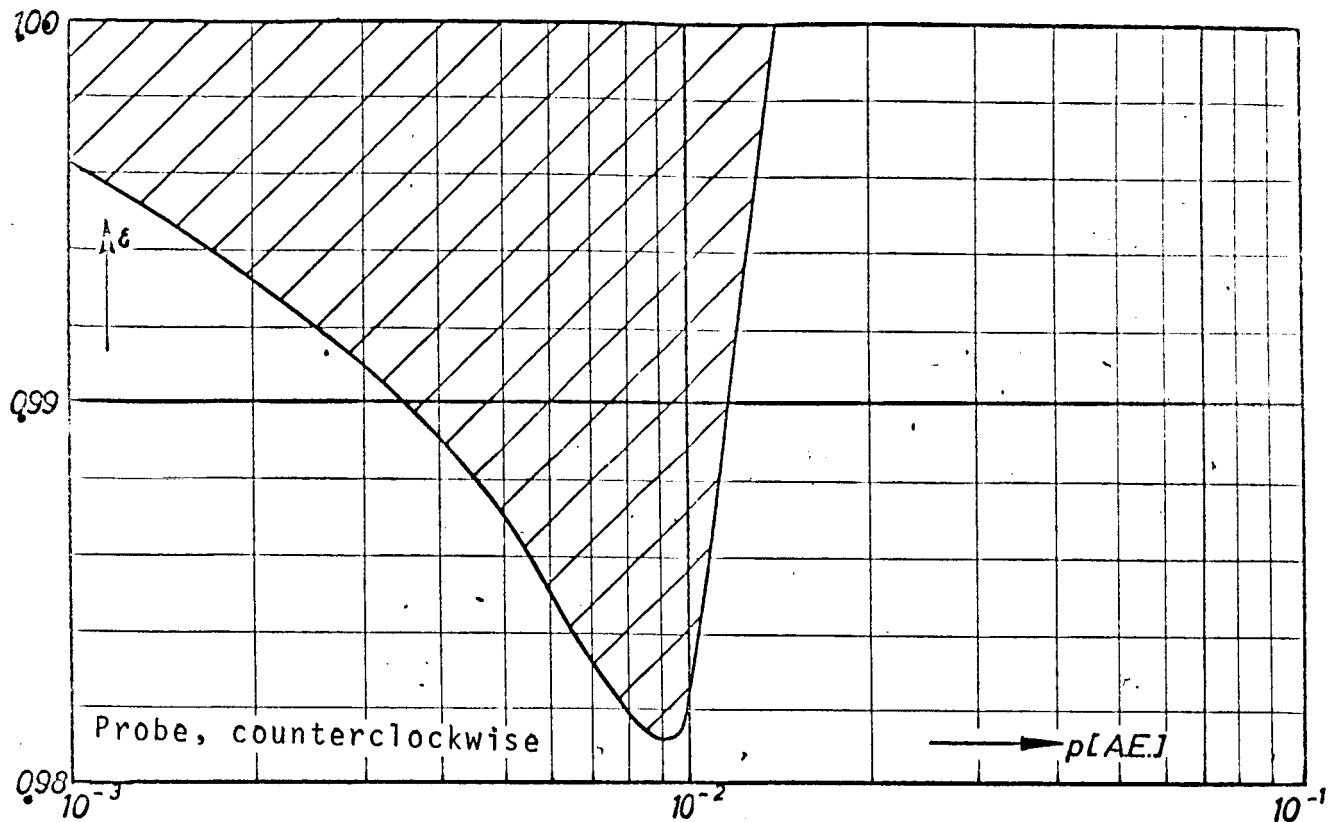


Fig. 10. Escape Behavior of the Jupiter/Ganymede System.

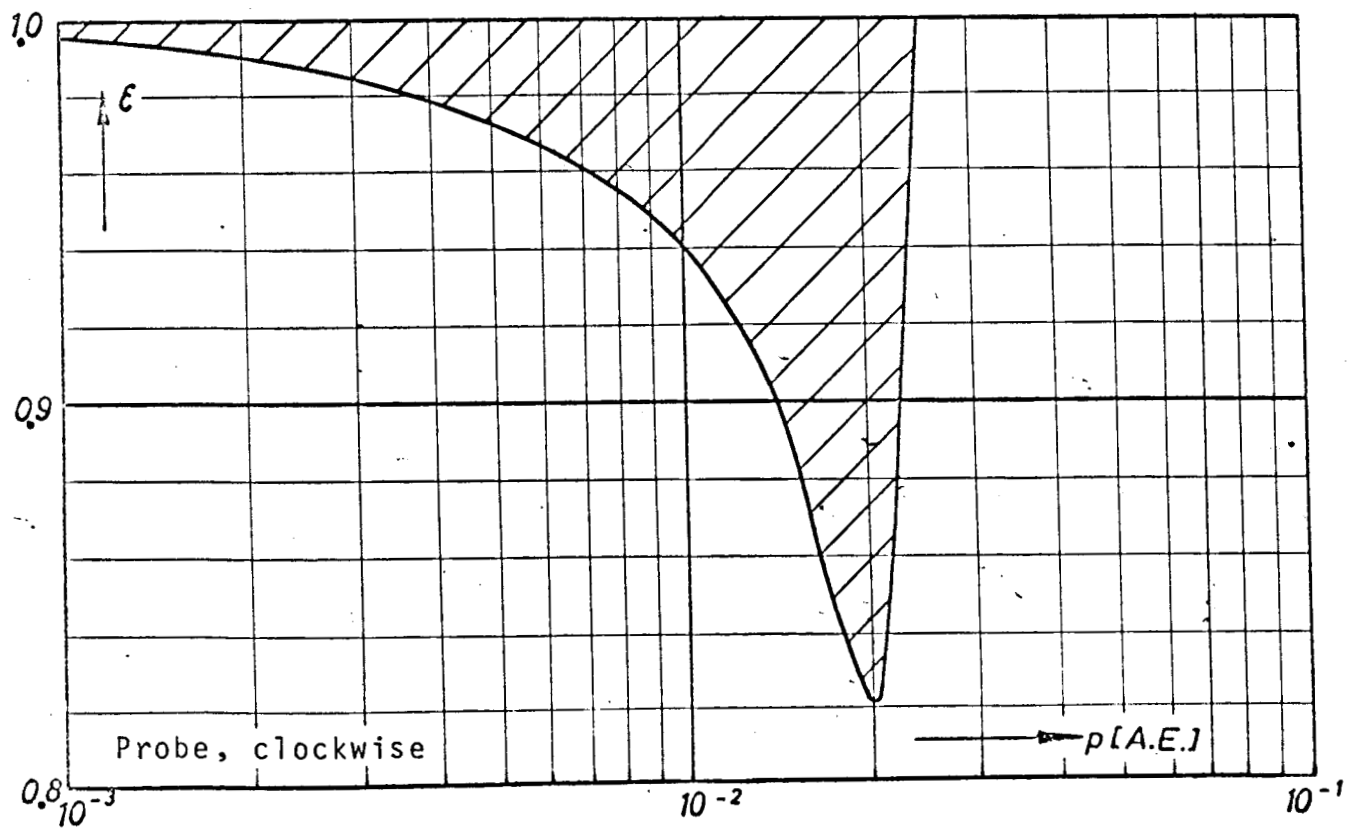
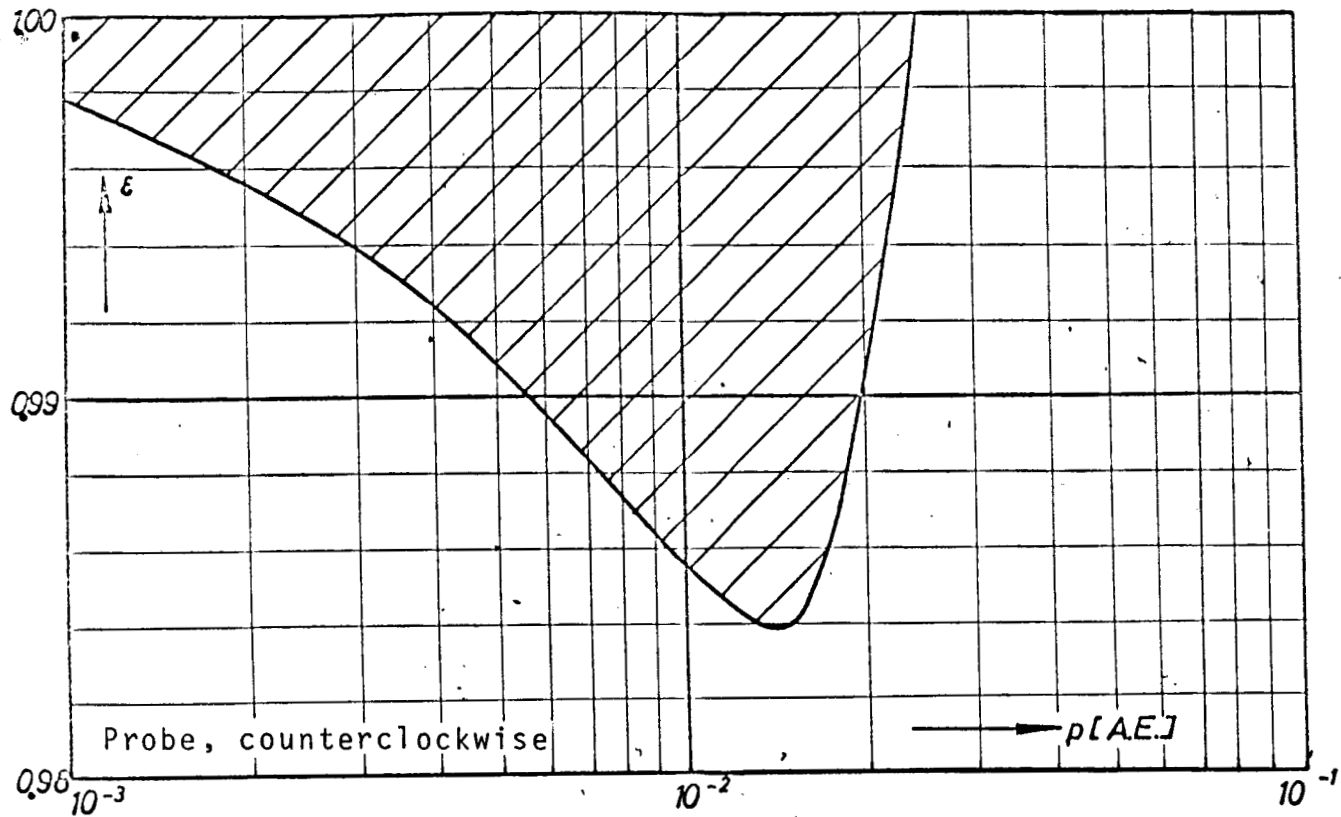


Fig. 11. Escape Behavior of the Jupiter-Callisto System

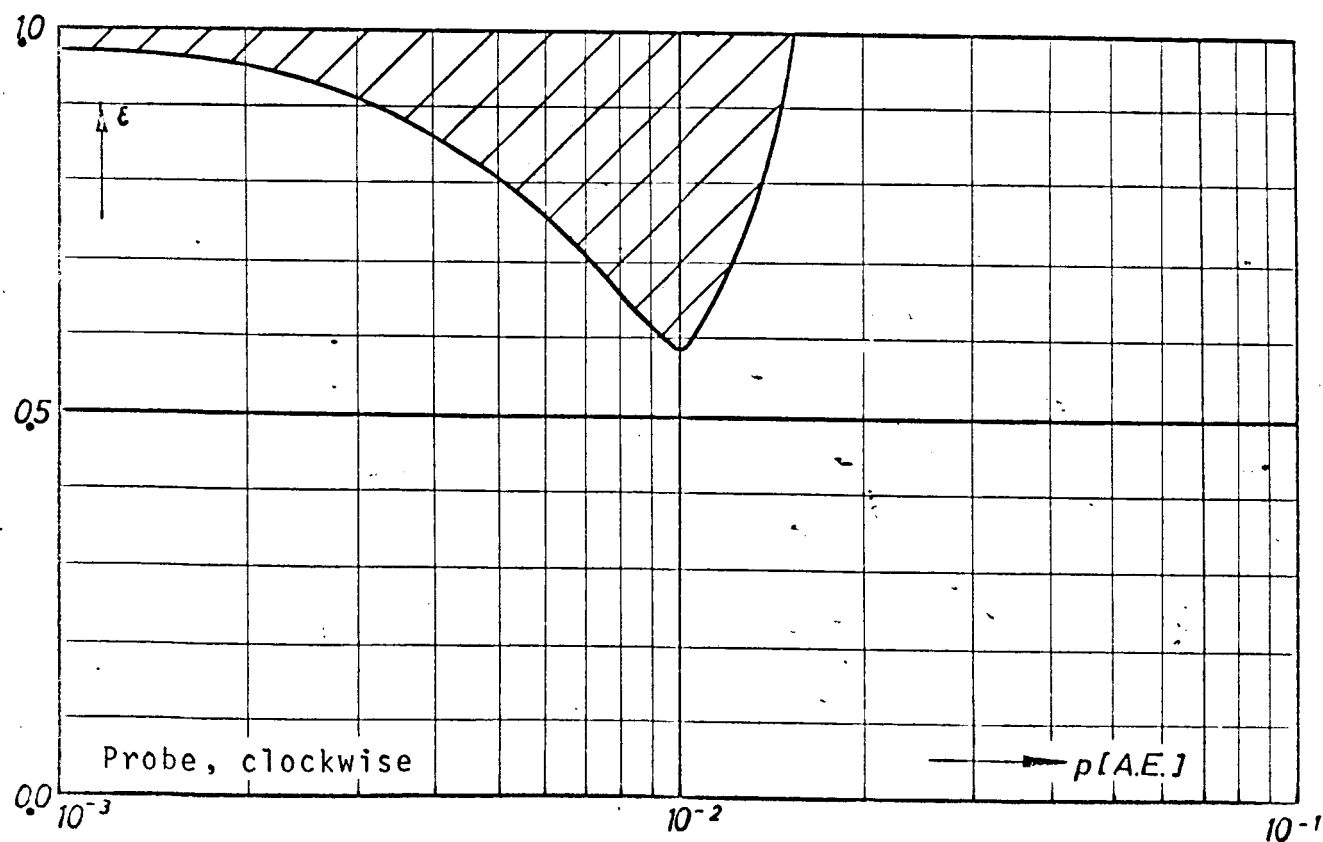
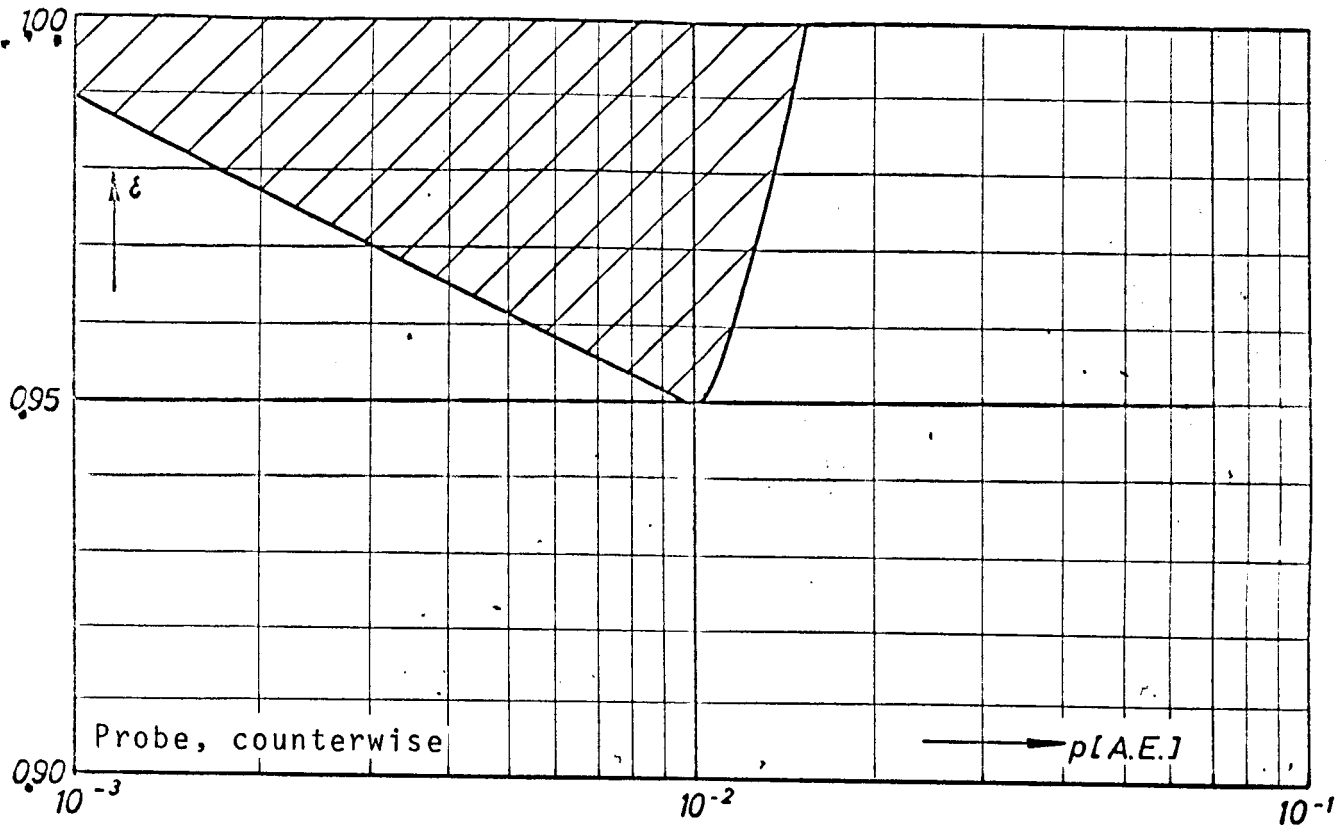


Fig. 12. Escape Behavior of the Saturn/Titan System

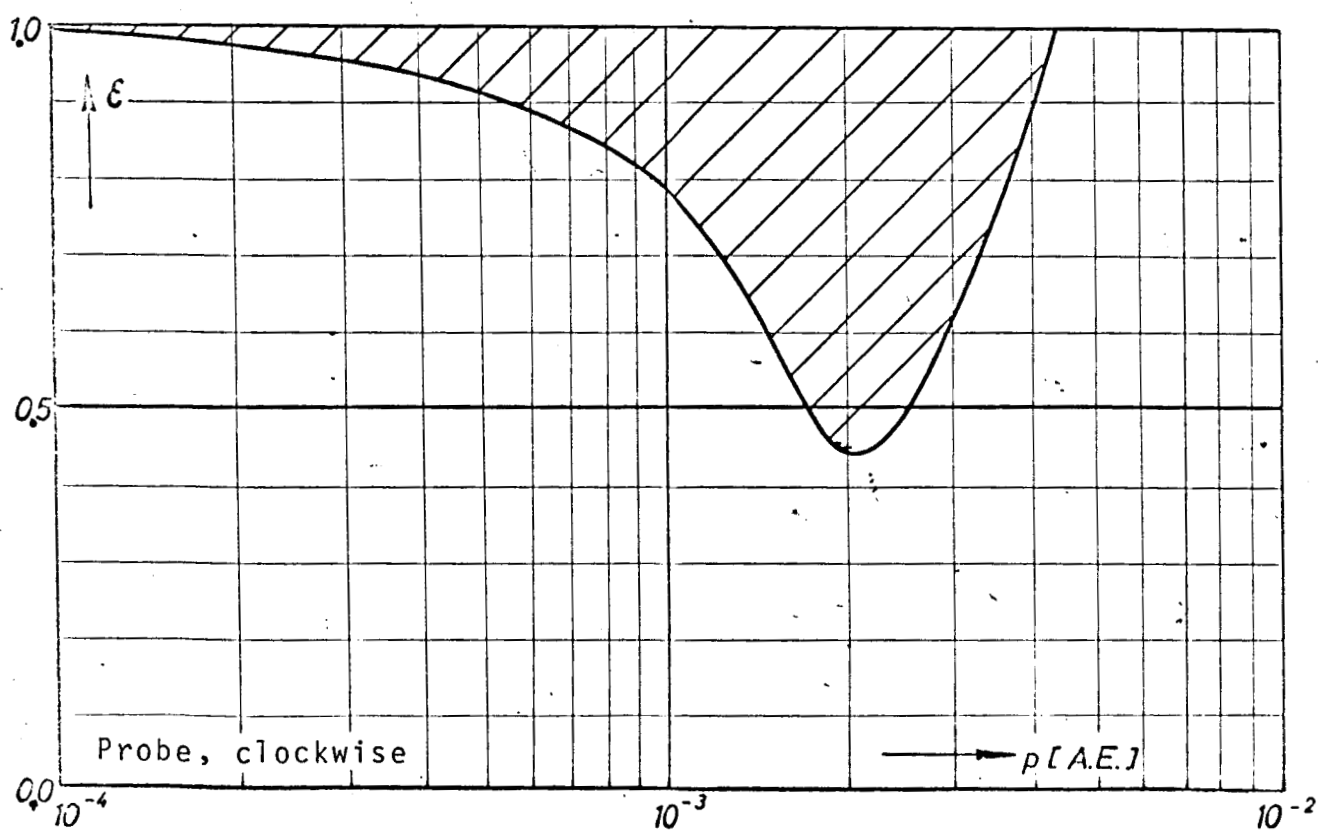
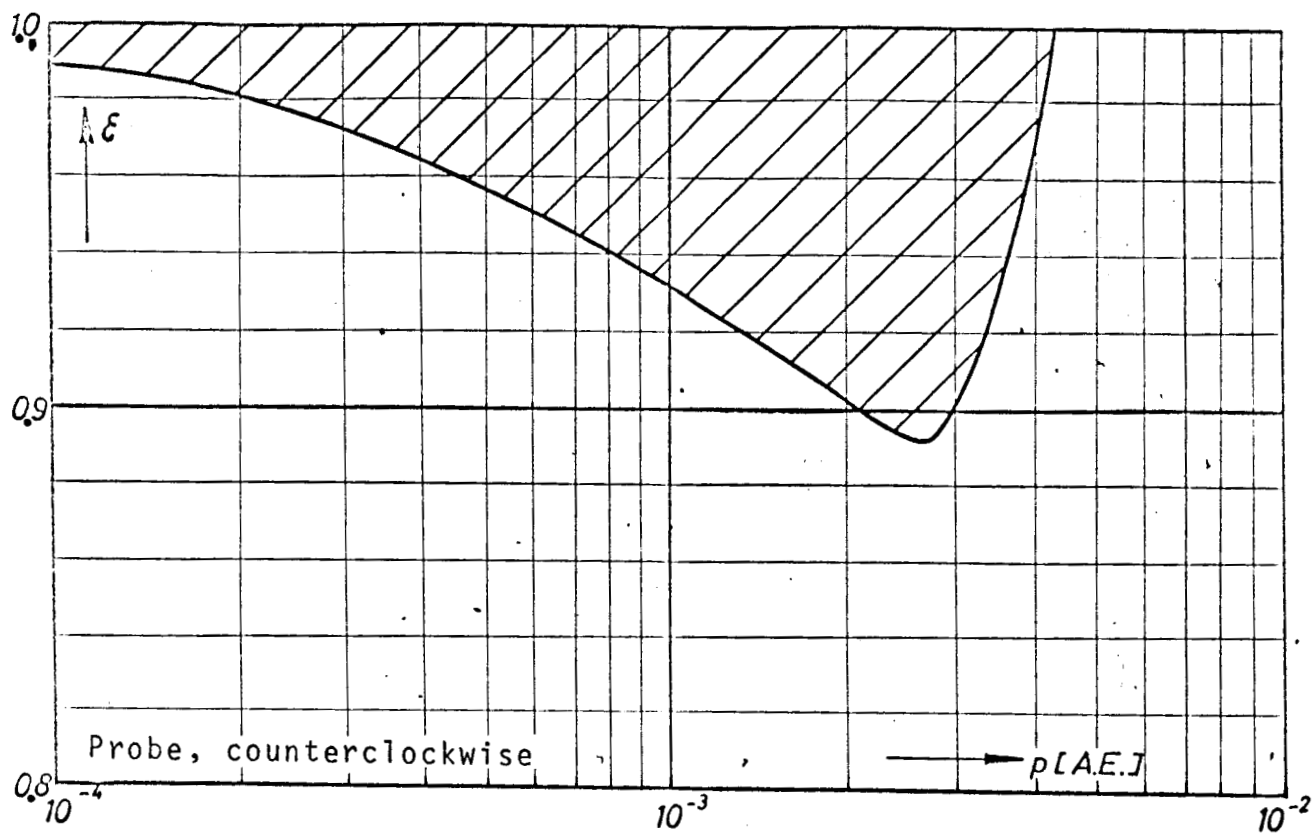
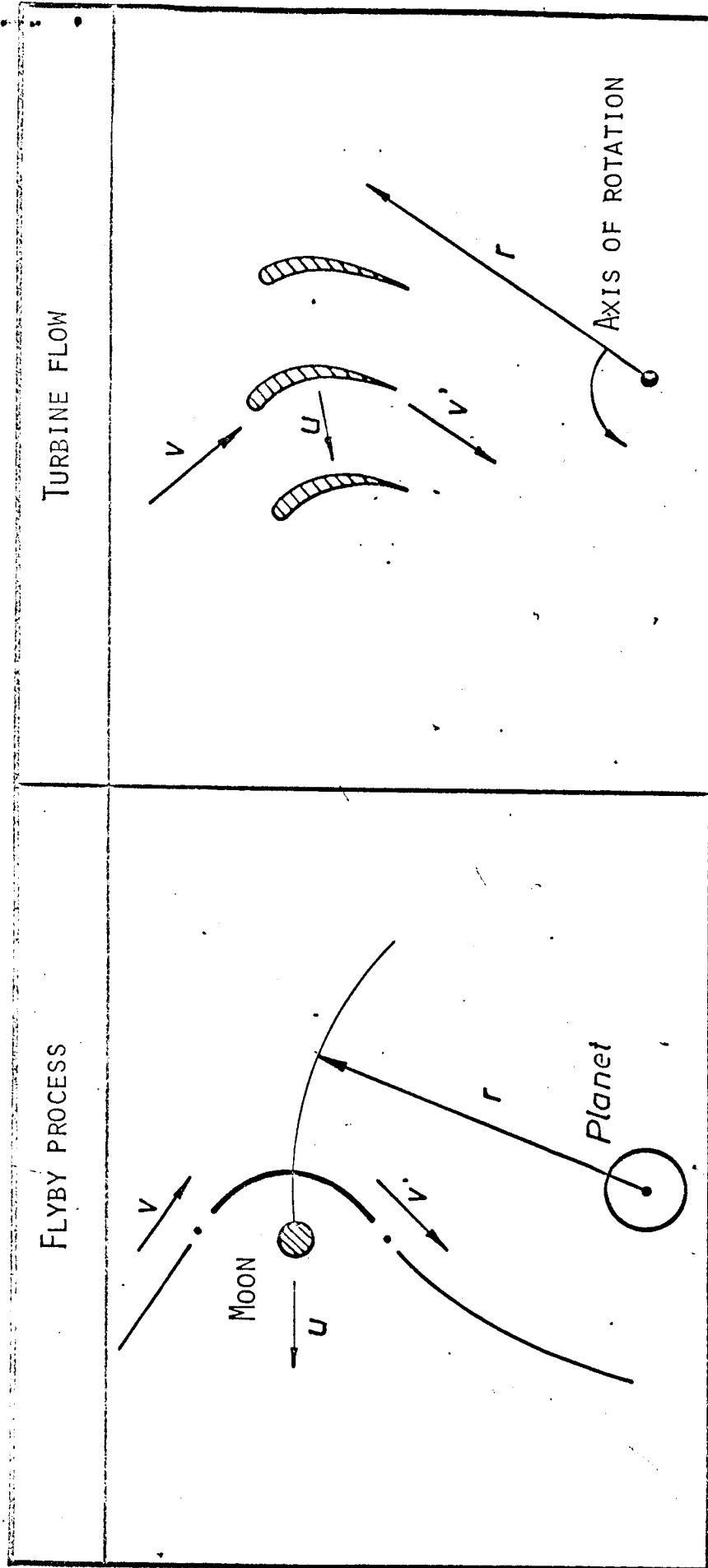


Fig. 13. Escape Behavior of the Neptune/Triton System



EULER TURBINE EQUATION: $\Delta E = \bar{u}(\bar{V} - \bar{v})$

DRALL-ALTERATION: $\Delta \bar{D} = \bar{r}(\bar{V} - \bar{v})$

Fig. 14. Analogy: Flyby Process-Turbine Flow

